Master in Actuarial Science

Risk Theory

27-06-2011

Time allowed: Three hours

Instructions:

1. This paper contains 7 questions and comprises 3 pages including the title page.
2. Enter all requested details on the cover sheet.
3. You have 10 minutes of reading time. You must not start writing your answers until instructed to do so.
4. Number the pages of the paper where you are going to write your answers.
5. Attempt all 7 questions.
6. Begin your answer to each of the 7 questions on a new page.
7. Marks are shown in brackets. Total marks: 200.
8. Show calculations where appropriate.
9. An approved calculator may be used.
10. The distributed Formulary and the Formulae and Tables for Actuarial Examinations (the 2002 edition) may be used. Note that the parametrization used for the different distributions is that of the distributed Formulary.

1. Let $\{N(t), t \geq 0\}$ be an homogeneous Poisson process with intensity λ. Let W_n denote the time of the n-th event. Determine, justifying

   (a) $E[W_5]$, [5]

   (b) $E[W_5 | N(1) = 2]$, [10]

   (c) $E[N(4) - N(2) | N(1) = 3]$. [5]

2. Consider that the number of accidents per year, N, of a policy, follows a mixed Poisson distribution with structure distribution Λ. Suppose that each time there is an accident, the policyholder reports a claim if the amount of the loss exceeds the deductible d. Let $F_X(x)$ be the distribution function of the losses, and suppose that they are independent, and independent of the number of accidents. Let M be the number of reported claims per year. Writing M as a compound mixed Poisson distribution, and using generating functions, show that M is still a mixed Poisson random variable. [20]

3. An insurance company feels that each of its policyholders in a given portfolio has a rating value and that a policyholder having rating value λ will make claims according to a Poisson process with rate λ, when time is measured in years. The company also believes that rating values vary from policyholder to policyholder, with the probability distribution of the value of a new policyholder being Gamma distributed with parameters $(2, 1)$. Calculate the probability that no claims have been reported by the end of year 3 for a randomly selected policy. [15]

4. There are two types of claims that are made to an insurance company. Let $N_i(t)$ denote the number of type i claims made by time t , and suppose that $\{N_1(t), t \geq 0\}$ and $\{N_2(t), t \geq 0\}$ are independent Poisson processes with rates $\lambda_1 = 100$ and $\lambda_2 = 1$ per year. The amounts of successive type 1 claims (in a given monetary unit) are independent exponential random variables with mean 1 whereas the amounts from type 2 claims (in the same monetary unit) are independent Pareto random variables with mean 1 and variance 3.

   Consider that the portfolio is reinsured by an excess of loss cover, on a annual basis, with a retention limit of 5 and that the company only reports to the reinsurer claims in excess of the retention limit. Consider that the reinsurance premium is calculated according to the standard deviation principle with loading coefficient 300%.

   (a) A claim has just been received; what is the probability it is a type 2 claim? [5]

   (b) A claim has been received. What is the survival function of its severity? [5]

   (c) A claim has been received. What is the probability that it is reported to the reinsurer? [5]

   (d) You know that a claim has just been received and that it will be reported to the reinsurer; what is the probability it is a type 2 claim? [10]

   (e) Determine the reinsurance premium. [10]

5. Consider a group life insurance scheme, for simplification with only four classes. In the following table you are given, for each of the classes, the number of lives, n_k, the benefit (insured capital), b_k, and the probability of a claim in a year, q_k.

k	n_k	q_k	b_k
1	20	0.01	1
2	10	0.01	2
3	30	0.02	1
4	40	0.02	2

Let N be the annual number of claims and S the respective aggregate claim amount.

(a) Considering that the 100 lives are independent, determine μ_S, σ_S and γ_S. [15]

(b) Calculate an approximation to the probability that S is greater than 6. [15]

(c) If N was approximated by a Poisson, with the same mean, determine the probability that S is greater than 3, using the recursion formula. [25]

6. Suppose that, in the classical model, the loading is 0.8 of the expected aggregate claims and that the claim size density is

$$f_X(x) = (1 + 6x)\exp(-3x), x > 0.$$

(a) Determine the ultimate ruin probability, $\psi(u)$. [20]

(b) What is the Lundberg's bound to the ultimate probability of ruin. [5]

7. Let X be a random variable with distribution function

$$F_X(x) = 1 - e^{-\frac{1}{2}x^2}, x > 0.$$

(a) Determine $VaR_{0.99}(X)$. [5]

(b) Show that $E[X \wedge d] = \sqrt{2\pi}\left(\Phi(d) - \frac{1}{2}\right)$, where $\Phi(.)$ is the distribution function of the standard normal. [15]

(c) For a deductible of 1 calculate the Loss Elimination Ratio. [5]

(d) Determine $TVaR_{0.99}(X)$. [5]